

## The universe out of a monopole in the laboratory? \*

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### Abstract

To explore the possibility that an inflationary universe can be created out of a stable particle in the laboratory, we consider the classical and quantum dynamics of a magnetic monopole in the thin-shell approximation. Classically there are three types of solutions: stable, collapsing and inflating monopoles. We argue that the transition from a stable monopole to an inflating one could occur either by collision with a domain wall or by quantum tunneling.

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## I. INTRODUCTION

For many years it has been discussed whether it is possible to create a universe in the laboratory [1–5]. The original idea was based on the model of a false-vacuum (de Sitter) bubble embedded in an asymptotically flat (Schwarzschild) spacetime. The classical dynamics of false-vacuum bubbles was originally studied by Sato, Sasaki, Kodama, and Maeda [6] in the context of old inflation [7], and investigated systematically by Blau, Guendelman, and Guth [8] with Israel’s junction conditions [9]. If a false-vacuum bubble is larger than the de Sitter horizon, the bubble inflates eternally. Because the inflating bubble is surrounded by black hole horizons and causally disconnected by the “original universe”, such a bubble is called a “child universe”.

Farhi and Guth [1] discussed whether such a false-vacuum bubble is created in the laboratory, applying the Penrose theorem [10]. The theorem states that, if

- (a) there exists a noncompact Cauchy surface,
- (b)  $\mathcal{R}_{\mu\nu}k^\mu k^\nu \geq 0$  for all null vector  $k^\mu$ ,
- (c) there exists an anti-trapped surface <sup>1</sup>,

then there exists at least one past incomplete null geodesic. As a consequence of the Einstein equations  $G_{\mu\nu} = 8\pi GT_{\mu\nu}$ , condition (b) is rewritten as  $T_{\mu\nu}k^\mu k^\nu \geq 0$ . Because any standard theory of matter, including a canonical scalar field, obeys this energy condition, we may conclude that it is impossible to create an inflationary universe in the laboratory. Condition (c) represents the realization of an inflationary universe since the existence of an anti-trapped surface means the existence of the cosmological horizon. To put it simply, a false-vacuum bubble large enough to be an inflationary universe cannot avoid an initial singularity, while a bubble without an initial singularity is too small to expand.

Because the above argument is based on the classical field theory, a quantum process could make it possible to produce a large false-vacuum bubble without an initial singularity. Actually, Farhi, Guth, and Guven [2] and Fischler, Morgan, and Polchinski [3] considered a quantum decay from a small bubble without an initial singularity to a large bubble which becomes an inflationary universe, and calculated its probability.

As Guendelman and Portnoy [4] pointed out, however, there is a problem in the model. Because the effective potential which governs the shell trajectories has no local minimum, there is no stable solution. Even if we succeed to make a small false-vacuum bubble, the bubble collapses as soon as it is created; there is almost no chance for a quantum decay to happen during its lifetime. To solve this problem, Guendelman and Portnoy proposed a new model. They assumed a (2+1)-dimensional gauge field localized on the surface of a false-vacuum bubble. Due to the gauge field, there exists a static and stable classical configuration, which eventually decays into an inflationary universe.

Guendelman and Portnoy [5] also proposed another model of a (2+1)-dimensional spacetime, where a massless scalar field localized at the (1+1)-dimensional boundary maintains a stable classical configuration. A new aspect of this model is that an inflationary universe can be created by an arbitrarily small tunneling, which they called “almost classical creation of a universe”.

In this paper we consider the possibility that a stable magnetic monopole evolves into an inflationary universe. In the Einstein-Yang-Mills-Higgs system static monopole solutions

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<sup>1</sup>An anti-trapped surface is defined as the spacelike closed two-surface such that the expansion of both sets (i.e., ingoing and outgoing) of future directed null orthogonal to the two-surface is everywhere positive. This is just the past trapped surface introduced by Hayward [11].

were intensively studied [12], and then dynamical solutions were also analyzed [13]; one of the important results there is that there are stable solutions as well as inflating solutions with the same model parameters. We therefore expect the scenario that a classically stable monopole eventually evolves into an inflationary universe.

Although magnetic monopoles have never been detected, unified theories of elementary particles predict their existence. Furthermore, monopole inflation [13–16], which is free from the fine-tuning problem of initial conditions and the graceful exit problem, is still viable. Therefore, the monopole model is more realistic and motivated than the previous models.

Specifically, we adopt the thin-shell model of Arreaga, Cho, and Guven [16] (except for the form of the surface density). A monopole is modeled as follows: the inside is de Sitter spacetime, the outside is Reissner-Nordström, and the boundary is a timelike hypersurface. Here we should remark the limitation of this approximation. Numerical analysis of monopole inflation with the potential  $V = (\lambda/4)(\Phi^2 - \eta^2)^2$  showed that the the boundary becomes spacelike once inflation begins [15]. Nevertheless, the thin-shell model give a reliable result when a monopole oscillates stably or just begins to expand. Because we are interested only in the transition from a stable state to an expanding state, the present model is effective.

The plan of this paper is as follows. In Sec. II we derive the classical action and the equation of motion of the shell. In Secs. III and IV we consider the possibility that a classically stable monopole evolves into an inflationary universe by classical processes (Sec. III) and by quantum tunneling (Sec. IV). Sec. V is devoted to summary and discussions. In this paper we use the units  $c = \hbar = 1$ , but occasionally we write  $\hbar$  explicitly.

## II. CLASSICAL ACTION AND EQUATIONS OF MOTION

The original model we consider is the SU(2) Einstein-Yang-Mills-Higgs system:

$$\mathcal{S} = \mathcal{S}_g + \mathcal{S}_m = \int d^4x \sqrt{-g} \left[ \frac{\mathcal{R}}{16\pi G} - \frac{1}{4}(F_{\mu\nu}^a)^2 - \frac{1}{2}(D_\mu \Phi^a)^2 - V(\Phi) \right], \quad (2.1)$$

with

$$V(\Phi) = \frac{1}{4}\lambda(\Phi^2 - \eta^2)^2, \quad \Phi \equiv \sqrt{\Phi^a \Phi^a}, \quad (2.2)$$

$$F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + e\epsilon^{abc} A_\mu^b A_\nu^c, \quad D_\mu \Phi^a \equiv \nabla_\mu \Phi^a + e\epsilon^{abc} A_\mu^b \Phi^c, \quad (2.3)$$

where  $A_\mu^a$  and  $F_{\mu\nu}^a$  are the SU(2) Yang-Mills field potential and its field strength, respectively.  $\Phi^a$  is the real triplet Higgs field, and  $V(\Phi)$  is its potential.  $\lambda$  and  $e$  are the Higgs self-coupling constant and the gauge coupling constant, respectively.  $\nabla_\mu$  and  $D_\mu$  are the spacetime covariant derivative and the totally covariant derivative, respectively. The variation of (2.1) with respect to  $g_{\mu\nu}$ ,  $\Phi^a$ , and  $A_i^a$  yields the Einstein equations

$$G_{\mu\nu} \equiv \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = 8\pi GT_{\mu\nu}, \quad (2.4)$$

$$T_{\mu\nu} \equiv D_\mu \Phi^a D_\nu \Phi^a - g_{\mu\nu} \left[ \frac{1}{2}(D_\sigma \Phi^a)^2 + V(\Phi) \right] + F_{\mu\lambda}^a F_\nu^{a\lambda} - \frac{1}{4}g_{\mu\nu}(F_{\lambda\sigma}^a)^2, \quad (2.5)$$

and the equations for the matter fields:

$$D_\mu D^\mu \Phi^a = \frac{\partial V(\Phi)}{\partial \Phi^a}, \quad (2.6)$$

$$D_\mu F^{a\mu\nu} = e\epsilon^{abc}\Phi^b D^\nu \Phi^c. \quad (2.7)$$

We assume a spherically symmetric spacetime and adopt the 't Hooft-Polyakov ansatz for the matter field:

$$\Phi^a = \Phi(x^0, x^1)\hat{r}^a, \quad \hat{r}^a \equiv (\sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta), \quad (2.8)$$

$$A_\mu^a = \epsilon^{abc}(\partial_\mu \hat{r}^b)\hat{r}^c \frac{1-w(x^0, x^1)}{e}. \quad (2.9)$$

The purpose of this section is to reduce the action (2.1) to its thin-shell limit and derive the equations of motion. We essentially follow Farhi *et al.* [2] and Ansoldi *et al.* [17] except for treatment of the boundary of the region of integration. We will not introduce the boundary term to cancel out the second derivatives of the metric; instead we will delete the second derivatives just by integration by part with fixing all dynamical variables and their first derivatives at the boundary. Because the two methods are equivalent, we choose this simple and straightforward way.

In the thin-shell limit, the outside ( $V_+$ ) and the inside ( $V_-$ ) are characterized by  $\Phi = \eta$ ,  $w = 0$  and  $\Phi = 0$ ,  $w = 1$ , respectively. Then, the spacetime solutions and the energy-momentum tensors of both sides are respectively given by

$$ds^2 = -A_+ dt_+^2 + \frac{dr_+^2}{A_+} + r_+^2(d\theta_+^2 + \sin^2\theta_+ d\varphi_+^2), \quad A_+(r_+) \equiv 1 - \frac{2GM}{r_+} + \frac{GQ^2}{r_+^2}. \quad (2.10)$$

$$T_\nu^{\mu+} = \frac{1}{2e^2 r_+^4} \text{diag}(-1, -1, 1, 1), \quad Q^2 = \frac{4\pi}{e^2} \quad (2.11)$$

$$ds^2 = -A_- dt_-^2 + \frac{dr_-^2}{A_-} + r_-^2(d\theta_-^2 + \sin^2\theta_- d\varphi_-^2), \quad A_-(r_-) \equiv 1 - H^2 r_-^2, \quad (2.12)$$

$$T_\nu^{\mu-} = -\rho \delta_\nu^\mu, \quad H^2 = \frac{8\pi G}{3}\rho, \quad (2.13)$$

where  $\rho = V(0) = \lambda\eta^4/4$  is a constant. The two regions are connected at the  $r_- = r_+ = R$  spherical hypersurface  $\Sigma$ . Because we can identify  $(r_-, \theta_-, \varphi_-)$  and  $(r_+, \theta_+, \varphi_+)$  on  $\Sigma$ , hereafter we omit the signs  $\pm$  in these coordinates.

The matter part (of the action) in  $V^\pm$  is evaluated as

$$\mathcal{S}_m^+ = \int_{t_i^+}^{t_f^+} dt_+ \int_R^\infty dr \, 4\pi r^2 \left( -\frac{1}{2e^2 r^4} \right) = - \int_{t_i^+}^{t_f^+} dt_+ \frac{Q^2}{2R}, \quad (2.14)$$

$$\mathcal{S}_m^- = \int_{t_i^-}^{t_f^-} dt_- \int_0^R dr \, 4\pi r^2 (-\rho) = - \int_{t_i^-}^{t_f^-} dt_- \frac{4\pi\rho}{3} R^3, \quad (2.15)$$

where  $[t_i^\pm, t_f^\pm]$  is the time interval under consideration. For the gravity part in  $V^\pm$ , we apply the Einstein equation (2.4), which reads  $\mathcal{R}^\pm = -8\pi G T^\pm$ . Then we find

$$\mathcal{S}_g^+ = 0, \quad \mathcal{S}_g^- = \int_{t_i^-}^{t_f^-} dt_- \int_0^R dr \, 4\pi r^2 \frac{-8\pi G}{16\pi G} (-4\rho) = \int_{t_i^-}^{t_f^-} dt_- \frac{8\pi\rho}{3} R^3. \quad (2.16)$$

To describe the geometry in the neighborhood of  $\Sigma$  we introduce the Gaussian normal coordinate system:

$$ds^2 = dn^2 + \gamma_{ij} dx^i dx^j = -\bar{N}(\tau, n)^2 d\tau^2 + dn^2 + \bar{R}(\tau, n)^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (2.17)$$

and define the metric functions on the shell ( $\Sigma : n = 0$ ) as

$$N(\tau) \equiv \bar{N}(\tau, 0), \quad R(\tau) \equiv \bar{R}(\tau, 0). \quad (2.18)$$

Following Ansoldi *et al.* [17], we keep  $N$  an arbitrary function to derive the constraint equation from the variational principle.

The matter part on  $\Sigma$  should be derived from the original action (2.1). We depart from Arreaga *et al.* [16] by considering the Yang-Mills gauge field as well. Because the dominant contribution on the shell comes from gradient energy, the matter part of the action approximates

$$\mathcal{S}_m \approx \int_{\tau_i}^{\tau_f} d\tau \int_{-0}^{+0} dn \, 4\pi \bar{N} \bar{R}^2 \left\{ -\frac{(\partial_n \Phi)^2}{2} - \left( \frac{\partial_n w}{e \bar{R}} \right)^2 \right\}. \quad (2.19)$$

Defining two functions of  $\tau$  (definite integrals of  $n$ ) as

$$\sigma_0 \equiv \int_{-0}^{+0} dn \, \frac{(\partial_n \Phi)^2}{2}, \quad \sigma_1 \equiv \int_{-0}^{+0} dn \, \frac{(\partial_n w)^2}{e^2}, \quad (2.20)$$

the action in the thin-shell limit becomes

$$\mathcal{S}_m^\Sigma = \int_{\tau_i}^{\tau_f} N d\tau (-4\pi \sigma R^2), \quad \sigma(\tau) \equiv \sigma_0(\tau) + \frac{\sigma_1(\tau)}{R^2(\tau)}. \quad (2.21)$$

To evaluate the gravity part for  $\Sigma$ , we define the extrinsic curvature tensor as

$$K_{ij} \equiv \nabla_j n_i = -\Gamma_{ij}^n = \frac{\partial_n \gamma_{ij}}{2} \quad \text{at } n \rightarrow \pm 0, \quad (2.22)$$

where  $n^\mu$  is the normal vector of  $\Sigma$  pointing outward, and given by  $n^n = 1$  and  $n^i = 0$  in the coordinate system (2.17). Then we can decompose the four dimensional Ricci scalar into the three dimensional Ricci scalar and the extrinsic curvature:

$$\begin{aligned} \mathcal{S}_g^\Sigma &= \int_{\tau_i}^{\tau_f} \bar{N} d\tau \int_{-0}^{+0} dn \, 4\pi \bar{R}^2 \frac{1}{16\pi G} (\mathcal{R}^{(3)} - K_{ij} K^{ij} - K^2 - 2\partial_n K) \\ &= \int_{\tau_i}^{\tau_f} N d\tau \left( -\frac{R^2}{2G} [K]^\pm \right), \end{aligned} \quad (2.23)$$

where  $K \equiv K_i^i$  and  $[K]^\pm \equiv K^+ - K^-$ . The components of  $K_{ij}$  for the line elements (2.12) and (2.10) are calculated as

$$K_\theta^{\theta\pm} = \frac{\beta^\pm}{R}, \quad \beta^\pm \equiv \partial_n \bar{R}^\pm = \varepsilon^\pm \sqrt{\dot{R}^2 + A_\pm(R)}, \quad (2.24)$$

$$K_\tau^{\tau\pm} = \frac{1}{\beta^\pm} \left( \ddot{R} + \frac{A'_\pm(R)}{2} \right), \quad (2.25)$$

where

$$\varepsilon^\pm = +1 \text{ or } -1, \quad \dot{\cdot} \equiv \frac{1}{N} \frac{d}{d\tau}, \quad ' \equiv \frac{d}{dr}. \quad (2.26)$$

From (2.14)-(2.16), (2.21) and (2.23), the total action is reduced to

$$\mathcal{S} = - \int_{t_i^+}^{t_f^+} dt_+ \frac{Q^2}{2R} + \int_{t_i^-}^{t_f^-} dt_- \frac{4\pi\rho}{3} R^3 - \int_{\tau_i}^{\tau_f} N d\tau \left\{ 4\pi\sigma R^2 + \frac{1}{G} \left[ \frac{R^2}{2\beta} \left( \ddot{R} + \frac{A'}{2} \right) + R\beta \right]^\pm \right\} \quad (2.27)$$

To remove the second-derivative term from the action, we integrate it by part (with respect to the proper time  $T \equiv \int N d\tau$ ):

$$\int_{\tau_i}^{\tau_f} N d\tau \frac{R^2 \ddot{R}}{2\beta} = \frac{R^2}{2} \tanh^{-1} \left( \frac{\dot{R}}{\beta} \right) \Big|_{T_i}^{T_f} + \int_{\tau_i}^{\tau_f} N d\tau \left\{ -R\dot{R} \tanh^{-1} \left( \frac{\dot{R}}{\beta} \right) + \frac{R^2 \dot{R}^2 A'}{4\beta A} \right\} \quad (2.28)$$

Because the first term in the RHS contains only variables at the boundary, we can ignore it in variation. We also note the relation,

$$\int_{\tau_i}^{\tau_f} N d\tau \frac{R^2 A'_+ \beta_+}{4A_+} = \int_{t_i^+}^{t_f^+} dt_+ \frac{R^2 A'_+}{4} = \frac{GM}{2} (t_f^+ - t_i^+) - \int_{t_i^+}^{t_f^+} dt_+ \frac{GQ^2}{2R}. \quad (2.29)$$

Again we can ignore the first term in the RHS in variation, and the second term there is canceled by the first term in (2.27). Thus we arrive at the final form of the action:

$$\mathcal{S} = \int_{\tau_i}^{\tau_f} d\tau \mathcal{L}, \quad \mathcal{L} \equiv -N \left\{ 4\pi\sigma R^2 + \frac{R}{G} \left[ \beta - \dot{R} \tanh^{-1} \left( \frac{\dot{R}}{\beta} \right) \right]^\pm \right\}. \quad (2.30)$$

Keeping in mind that  $N$  is hidden in  $\dot{R} \equiv (dR/d\tau)/N$ , the variation of (2.30) with respect to  $N$  and  $R$  yields the classical equations of motion:

$$[K_\theta]^\pm \equiv \frac{[\beta]^\pm}{R} = -4\pi G \left( \sigma_0 + \frac{\sigma_1}{R^2} \right), \quad (2.31)$$

$$[K_\tau]^\pm \equiv \frac{[\dot{\beta}]^\pm}{\dot{R}} = -4\pi G \left( \sigma_0 - \frac{\sigma_1}{R^2} \right). \quad (2.32)$$

These equations give the energy-momentum conservation on the shell,

$$\dot{\sigma}_0 + \frac{\dot{\sigma}_1}{R^2} = 0. \quad (2.33)$$

The original field equations (2.6) and (2.7), however, give further conditions,

$$\dot{\sigma}_0 = \dot{\sigma}_1 = 0, \quad (2.34)$$

as is shown in Appendix.

Now we define the conjugate momentum as

$$P \equiv \frac{\partial \mathcal{L}}{\partial \dot{R}} = \frac{NR}{G} \left[ \tanh^{-1} \left( \frac{\dot{R}}{\beta} \right) \right]^\pm, \quad (2.35)$$

and the Hamiltonian as

$$\mathcal{H} \equiv P\dot{R} - \mathcal{L} = \frac{NR}{G}(\beta_+ - \beta_- + 4\pi G\sigma R). \quad (2.36)$$

The equations of motion (2.31) and (2.32) are equivalent to the Hamiltonian constraint:

$$\mathcal{H} = 0 \quad \text{with} \quad \sigma_0, \sigma_1 = \text{const.} \quad (2.37)$$

Formally one should invert (2.35) to express  $\dot{R}$  in terms of  $P$  in order to eliminate  $\dot{R}$  in (2.36), as Ansoldi *et al.* did [17]. In Sec. IV, however, we will quantize the system without the explicit form of  $\mathcal{H}(P, R)$ .

### III. CLASSICAL DYNAMICS

Hereafter we take  $N = 1$ . The classical motion is governed by the constraint equation (2.37), or (2.31). Following Arreaga *et al.* [16], we introduce dimensionless quantities,

$$\tilde{R} \equiv HR, \quad \tilde{\tau} \equiv H\tau, \quad m \equiv HGM, \quad q^2 \equiv H^2GQ^2, \quad s_0 \equiv \frac{4\pi G\sigma_0}{H}, \quad s_1 \equiv 4\pi GH\sigma_1, \quad (3.1)$$

to rewrite Eq.(2.31) as

$$\left(\frac{d\tilde{R}}{d\tilde{\tau}}\right)^2 + U(\tilde{R}) = -1, \quad (3.2)$$

$$U(\tilde{R}) \equiv -\left(\frac{1-s_0^2}{2}\tilde{R} - \frac{s_0s_1}{\tilde{R}} - \frac{m}{\tilde{R}^2} + \frac{q^2-s_1^2}{2\tilde{R}^3}\right)^2 \left(s_0 + \frac{s_1}{\tilde{R}^2}\right)^{-2} - \tilde{R}^2. \quad (3.3)$$

To understand the global spacetime structure, it is helpful to know the positions of horizons and the signs of  $\beta^\pm = \partial_n r_\pm$  in terms of  $\tilde{R}$ . De Sitter horizons  $\tilde{R}_D$ , the black-hole outer horizons  $\tilde{R}_{(+)}$  and the inner horizons  $\tilde{R}_{(-)}$  are characterized by

$$\tilde{R}_D = 1, \quad \tilde{R}_{(\pm)} = m \pm \sqrt{m^2 - q^2}, \quad (3.4)$$

respectively. To clarify the signs of  $\beta^\pm$ , using (2.31), we reexpress them as

$$\beta^\pm = \left(\mp \frac{s_0^2 - 1}{2}\tilde{R} \mp \frac{s_0s_1}{\tilde{R}} + \frac{m}{\tilde{R}^2} - \frac{q^2 \pm s_1^2}{2\tilde{R}^3}\right) \left(s_0 + \frac{s_1}{\tilde{R}^2}\right)^{-1}. \quad (3.5)$$

The conformal diagrams for de Sitter spacetime and Reissner-Nordström spacetime are shown in Fig. 1. We shall describe monopole solutions by joining a part of de Sitter spacetime to a part of Reissner-Nordström. The boundary of the two parts represents the trajectory of the shell  $\Sigma$ . We choose the monopole center as the “left”  $r = 0$  line in the diagram (a). Accordingly, the normal vector  $n^\mu$  of  $\Sigma$  points to the right in all diagrams. In the region  $r_- < r_D$  ( $r_+ > r_{(+)}$ ) the sign of  $\beta^- = \partial_n r_-$  ( $\beta^+ = \partial_n r_+$ ) is definite regardless of the shell motion. Inversely, the signs of (3.5) tell us approximate trajectories of the shell without solving the equation of motion (3.2).

There are four parameters,  $m$ ,  $q$ ,  $s_0$ , and  $s_1$ , which should be determined by the model parameters in (2.1) and initial conditions. From the field equations derived from (2.1), we can estimate their order-of-magnitude as

$$q^2 \sim \frac{\lambda}{e^2} \left( \frac{\eta}{m_{Pl}} \right)^4, \quad s_0 \sim \frac{\eta}{m_{Pl}}, \quad s_1 \sim \frac{\sqrt{\lambda}}{e} \left( \frac{\eta}{m_{Pl}} \right)^3, \quad (3.6)$$

where  $m_{Pl} \equiv 1/\sqrt{G}$  is the Planck mass. For example, if we assume  $\eta/m_{Pl} \sim 1$  and  $\lambda/e^2 \sim 1$ , all of them should be of order of unity. The precise values of those parameters cannot be determined without solving the field equations by fixing  $\Phi$  and  $\partial_t\Phi$  at  $t = 0$ ; there remains some ambiguity. We therefore regard the four parameters as free parameters in the range of order of unity. Here we do not survey all classical solutions, but only show some of them and discuss whether stable monopoles can evolve into inflating monopoles without an initial singularity, where we mean ‘‘initial singularity’’ by the spacetime singularity which exists in the past of the experimenter who makes an inflating monopole.

Figure 2 shows a classically stable oscillating monopole in a horizonless spacetime (type A) with  $m = 0.58$ ,  $q = 0.6$ ,  $s_0 = 0.6$  and  $s_1 = 0.1$ . In the same potential there is another expanding solution for large  $\hat{R}$  (type A’). Static and stable solutions obtained without thin-shell approximation [12] correspond to Type A.

If we increase  $m$ , the feature of solutions changes drastically. Figure 3 shows an inflating monopole (type B) with  $m = 0.64$ ,  $q = 0.6$ ,  $s_0 = 0.6$  and  $s_1 = 0.1$ . This phenomenon is consistent with the previous result [12,13] that static monopole solutions are nonexistent if their gravitational mass are large enough.

We expect that type A monopoles can evolve into type B by accretion of mass to the monopole. Specifically, we consider the model that a spherical domain wall surrounding the monopole eventually collides with it. Possible trajectories before and after the collision is shown in Fig. 4. This could be a classical process that an inflationary universe is created in the laboratory.

What about an initial singularity? In agreement with Farhi and Guth, this created universe includes past incomplete null geodesics emanating from anti-trapped surfaces, as is shown in Fig. 4. As one can easily see from Fig. 4, however, there is no initial singularity such as the Big Bang. Although a singularity exists in the past of the inflating monopole, the singularity is located in the future of the experimenter in the laboratory. In other words, even if no singularity exists in the past of the experimenter who makes a monopole, inflation in the monopole is realizable in the future of the experimenter. From an observational point of view, however, since the inflating monopole is realized inside a black hole, the experimenter cannot observe it unless he or she enters into the black hole. The detectability and stability of this solution will be discussed in the final section.

In the intermediate case between A and C solutions, there are classically stable oscillating solutions with black-hole horizons. Figure 5 shows the case of  $m = 0.61$ ,  $q = 0.6$ ,  $s_0 = 0.6$  and  $s_1 = 0.1$ . In this case there are two types of classical solutions: a stable oscillating monopole (type C) and an inflating monopole (type C’). Type C solutions are stable but do not fall into any solution in the study of static solutions [12]. It was found that stably oscillating solutions exist in the parameter range where static solutions are nonexistent [13]. Quantum tunneling from C to C’ is the subject of the next section.

#### IV. QUANTUM TUNNELING

To quantize the system we define the operators as

$$\hat{R} \equiv R, \quad \hat{P} \equiv i\hbar \frac{\partial}{\partial R}, \quad \hat{\mathcal{H}} \equiv \mathcal{H}(\hat{P}, \hat{R}). \quad (4.1)$$

and impose the Hamiltonian constraint on the quantum state  $\Psi$ ,



$$\hat{\mathcal{H}}(\hat{P}, \hat{R})\Psi(R) = 0. \quad (4.2)$$

If we write the wave function as

$$\Psi(R) = e^{iF(R)/\hbar}, \quad (4.3)$$

and substitute into (4.2), to lowest order in the WKB expansion, we obtain the Hamilton-Jacobi equation:

$$\mathcal{H}\left(\frac{dF}{dR}, R\right) = 0. \quad (4.4)$$

The solution  $F(R)$  is given by

$$F(R) = \int^R P(R)dR = \int^\tau d\tau R\dot{R} \left[ \tanh^{-1}\left(\frac{\dot{R}}{\beta}\right) \right]^\pm, \quad (4.5)$$

which is nothing but the action (2.30) with the Hamiltonian constraint  $\mathcal{H} = 0$ .

In the classically forbidden region we assume that there is a solution  $R(\tau_E)$  to the classical equation of motion, where  $\tau_E$  is the Euclidean time defined as

$$\tau_E \equiv i\tau. \quad (4.6)$$

We also define the Euclidean action as

$$F_E(R(\tau_E)) \equiv iF(R(i\tau)) = \int^{\tau_E} d\tau_E R \frac{dR}{d\tau_E} \left[ \tan^{-1}\left(\frac{dR/d\tau_E}{\beta_E}\right) \right]^\pm, \quad (4.7)$$

where

$$\beta_E \equiv \beta\left(\tau \rightarrow \frac{\tau_E}{i}\right) = \varepsilon \sqrt{-\left(\frac{dR}{d\tau_E}\right)^2 + A}. \quad (4.8)$$

Then the ratio of amplitudes at  $R_i$  and  $R_f$  is given by

$$\frac{\Psi(R_f)}{\Psi(R_i)} \approx \exp\left(-\frac{B}{\hbar}\right), \quad (4.9)$$

$$B \equiv F_E(R_f) - F_E(R_i) = \frac{1}{G} \int_{\tau_E^i}^{\tau_E^f} d\tau_E R \frac{dR}{d\tau_E} \left[ \tan^{-1}\left(\frac{dR/d\tau_E}{\beta_E}\right) \right]^\pm. \quad (4.10)$$

To integrate (4.10) we need to solve the Euclidean equation of motion, which is the analytic continuation of (2.31):

$$\beta_E^+ - \beta_E^- = -4\pi G\sigma R, \quad (4.11)$$

Rescaling the quantities as (3.1) and  $\tilde{\tau}_E \equiv H\tau$ , we can rewrite (4.11) as

$$\left(\frac{d\tilde{R}}{d\tilde{\tau}_E}\right)^2 = U(\tilde{R}) + 1. \quad (4.12)$$

In numerical calculation, it is more convenient to integrate its derivative,

$$\frac{d^2\tilde{R}}{d\tilde{\tau}_E^2} = \frac{1}{2} \frac{dU}{d\tilde{R}}, \quad (4.13)$$

once initial values are given by (4.12). The Euclidean junction condition (4.11) also gives

$$\beta_E^\pm = \left( \frac{\mp s_0^2 - 1}{2} \tilde{R} \mp \frac{s_0 s_1}{\tilde{R}} + \frac{m}{\tilde{R}^2} - \frac{q^2 \pm s_1^2}{2\tilde{R}^3} \right) \left( s_0 + \frac{s_1}{\tilde{R}^2} \right)^{-1}. \quad (4.14)$$

which is identical to the expression (3.5). The coefficient  $B$  is also rewritten with the normalized variables as

$$B = \frac{1}{GH^2} \int_{\tilde{\tau}_E^i}^{\tilde{\tau}_E^f} d\tilde{\tau}_E \tilde{R} \frac{d\tilde{R}}{d\tilde{\tau}_E} \left[ \tan^{-1} \left( \frac{d\tilde{R}/d\tilde{\tau}_E}{\beta_E} \right) \right]^\pm, \quad (4.15)$$

The method of numerical calculation of  $B$  is as follows. (i) Give the initial values  $\tilde{R}$  and  $d\tilde{R}/d\tilde{\tau}_E = 0$  (at  $\tilde{\tau} = \tilde{\tau}_E^i$ ), which satisfies (4.12). (ii) Integrate (4.13) and (4.15) with (4.14) until  $d\tilde{R}/d\tilde{\tau}_E = 0$  again (at  $\tilde{\tau} = \tilde{\tau}_E^f$ ). (iii) Check the accuracy of numerical solutions with (4.12).

Now let us investigate quantum tunneling from a classically stable monopole to inflating one. The first candidate is a quantum decay from type A to A'; however, it turns out to be impossible. Because  $\beta_+$  is always negative in type A', a possible spacetime structure is given by Fig. 2(c); the direction of the outer Reissner-Nordström region is opposite to that of type A in Fig. 2(b). It seems impossible that the outer infinite region becomes finite with a naked singularity by quantum tunneling. Actually, because a part of the action, given by (2.1), diverges if the outer region contains  $r = 0$ ,  $B$  also diverges and the probability  $\sim e^{-B}$  becomes zero.

On the other hand, the tunneling from type C to C' could happen. Therefore, we calculate  $B$  for this decay. Figure 6 shows some of the results, which are normalized by  $1/GH^2$ . The normalization factor is given by  $1/GH^2 = (3/2\pi\lambda)(m_{Pl}/\eta)^4$  in the model (2.1). Therefore, if  $\lambda\eta^4$  is not much smaller than the Planck density,  $B$  is not much larger than unity. In this case, the probability  $e^{-B}$  is considerable and the tunneling from type C to C' is likely to happen in the laboratory.

## V. SUMMARY AND DISCUSSIONS

To explore the possibility that an inflationary universe can be created out of a stable particle in the laboratory, we have considered the classical and quantum dynamics of a magnetic monopole in the thin-shell approximation.

There are two advantages in the monopole model, compared with the false-vacuum model. First, magnetic monopoles are natural consequences of particle theories; monopole inflation is also one of the consequences and still viable in cosmology. Second, contrary to the model of a false-vacuum bubble, there are stable solutions besides inflating and collapsing solutions in the present model. This is a preferable feature for making a universe in real experiments.

It has been believed that, as Farhi and Guth argued, the Penrose theorem indicates impossibility of creation of an inflationary universe without an initial singularity, and hence quantum tunneling has been devised to escape from the theorem. We have found, however, that in agreement with Farhi and Guth, there are past incomplete null geodesics, but the inflating monopole could be created by the experimenter whose past is geodesically complete.

We have proposed a specific model that a domain wall surrounding the stable monopole coalesces with it and becomes an inflating monopole.

For the trajectories beyond the inner horizon, or the Cauchy horizon, we should discuss instability of the horizon. Poisson and Israel [18] argued that, if radial perturbations are given, the gravitational mass inside the Cauchy horizon increases infinitely, which leads the appearance of a spacelike singularity near the Cauchy horizon. If perturbations are given in the present model for some physical reasons, classical solutions beyond the Cauchy horizon may break down. According to Dafermos [19], however, the spacetime could be extendible beyond such singularity as a  $C^0$  metric. Therefore, a physical consequence of our classical solutions is still unclear.

We have also analyzed a quantum decay from a classically stable monopole to an inflating one. We have adopted the canonical quantization of Farhi *et al.* to evaluate the probability amplitude to lowest order in WKB approximation. We find that, if the energy scale of the model is close to the Planck scale, the probability amplitude is considerable and the tunneling from type C to C' is likely to happen in the laboratory.

A problem of this model and other related models is the difficulty of detecting an inflationary universe because it is surrounded by an event horizon, which eventually disappears by Hawking radiation. Recently, Hawking [20] argued that information is preserved in black hole formation and evaporation, and information could get out of a black hole by radiation. Although this conjecture is uncertain at the moment, we expect that it will be a clue to detect a universe in the laboratory.

## ACKNOWLEDGMENTS

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## APPENDIX A: PROOF OF (2.34)

The field equations (2.6) and (2.7) with the Gaussian normal coordinates (2.17) are written as

$$\frac{1}{\bar{N}\bar{R}^2}\partial_\tau\left(\frac{\bar{R}^2}{\bar{N}}\partial_\tau\Phi\right) - \frac{1}{\bar{N}\bar{R}^2}\partial_n(\bar{N}\bar{R}^2\partial_n\Phi) + \frac{2}{\bar{R}^2}w^2\Phi + \frac{dV}{d\Phi} = 0, \quad (\text{A1})$$

$$\frac{1}{\bar{N}}\partial_\tau\left(\frac{\partial_\tau w}{\bar{N}}\right) - \frac{1}{\bar{N}}\partial_n(\bar{N}\partial_n w) + e^2\Phi^2 w + \frac{w(w^2 - 1)}{\bar{R}^2} = 0. \quad (\text{A2})$$

Multiplying (A1) by  $\partial_\tau\Phi$  and (A2) by  $\partial_\tau w$ , it follows that

$$\frac{1}{\bar{R}^4}\partial_\tau\left\{\frac{1}{2}\left(\frac{\bar{R}^2}{\bar{N}}\partial_\tau\Phi\right)^2\right\} + \partial_\tau\left\{\frac{(\partial_n\Phi)^2}{2}\right\} - \frac{\partial_n(\bar{N}\bar{R}^2\partial_\tau\Phi\partial_n\Phi)}{\bar{N}\bar{R}^2} + \partial_\tau\Phi\left(\frac{2}{\bar{R}^2}w^2\Phi + \frac{dV}{d\Phi}\right) = 0, \quad (\text{A3})$$

$$\partial_\tau\left\{\frac{1}{2}\left(\frac{\partial_\tau w}{\bar{N}}\right)^2 + \frac{(\partial_n w)^2}{2}\right\} - \frac{\partial_n(\bar{N}\partial_\tau w\partial_n w)}{\bar{N}} + w\partial_\tau w\left(e^2\Phi^2 + \frac{w^2 - 1}{\bar{R}^2}\right) = 0. \quad (\text{A4})$$

In the thin-shell limit we have assumed

$$\frac{(\partial_n \Phi)^2}{2} = \sigma_0(\tau) \delta(n), \quad (\partial_n w)^2 = e^2 \sigma_1(\tau) \delta(n), \quad (\text{A5})$$

while  $\Phi$ ,  $\partial_\tau \Phi$ ,  $w$  and  $\partial_\tau w$  are finite on the shell. Therefore, the integration of (A3) and (A4) with  $\int_{-0}^{+0} dn$  yields

$$\frac{d\sigma_0}{d\tau} - [\partial_\tau \Phi \partial_n \Phi]^\pm + \int_{-0}^{+0} dn \frac{\partial_n(\bar{N} \bar{R}^2)}{\bar{N} \bar{R}^2} \partial_\tau \Phi \partial_n \Phi = 0, \quad (\text{A6})$$

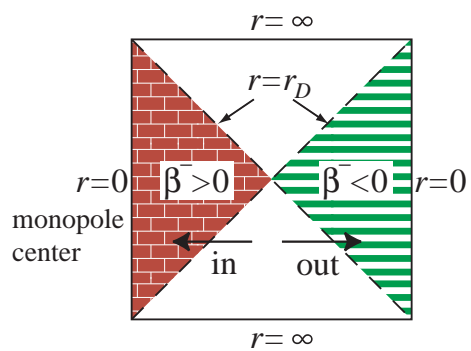
$$\frac{e^2}{2} \frac{d\sigma_1}{d\tau} - [\partial_\tau w \partial_n w]^\pm + \int_{-0}^{+0} dn \frac{\partial_n \bar{N}}{\bar{N}} \partial_\tau w \partial_n w = 0. \quad (\text{A7})$$

Although  $\partial_n \Phi$  and  $\partial_n w$  diverge at  $n = 0$ , their linear terms make no contribution to the integral because of  $\int_{-0}^{+0} dn \sqrt{\delta(n)} = 0$ . Hence, the third terms in (A6) and in (A7) vanish. Furthermore, because  $\Phi^\pm$  and  $w^\pm$  are constant in the present model, the second terms in (A6) and in (A7) also vanish. Thus (2.34) has been proved.

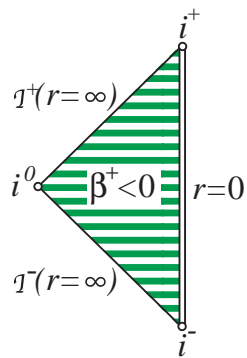
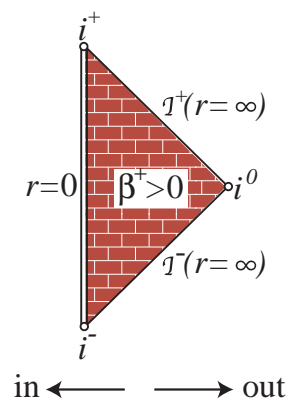
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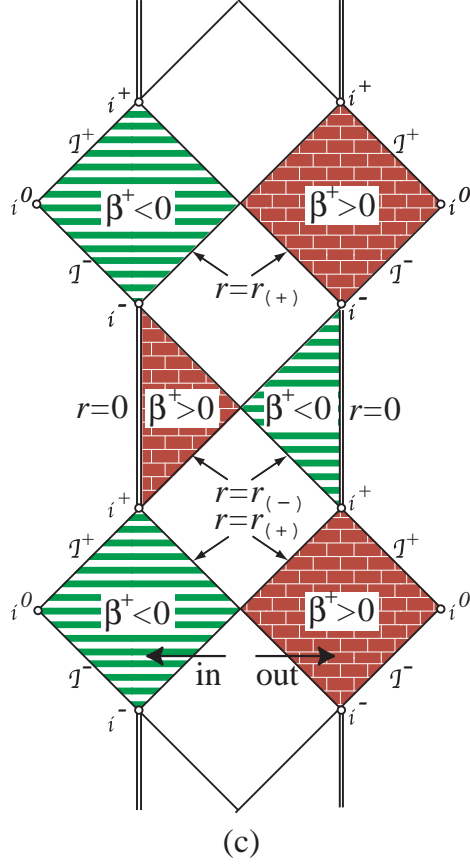
FIGURES



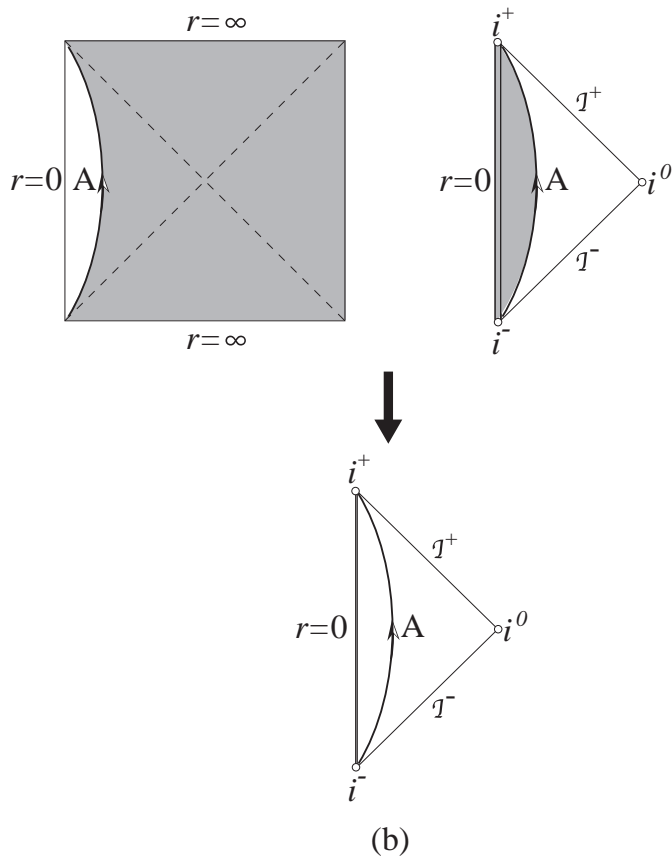
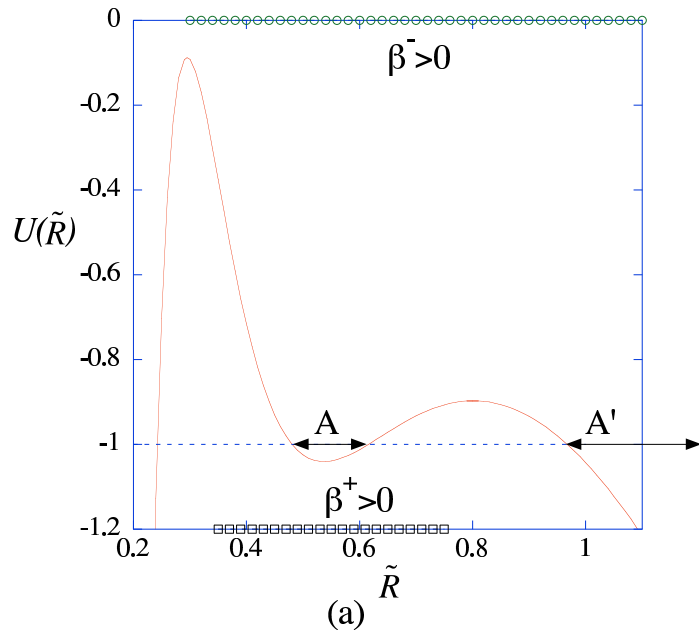
(a)



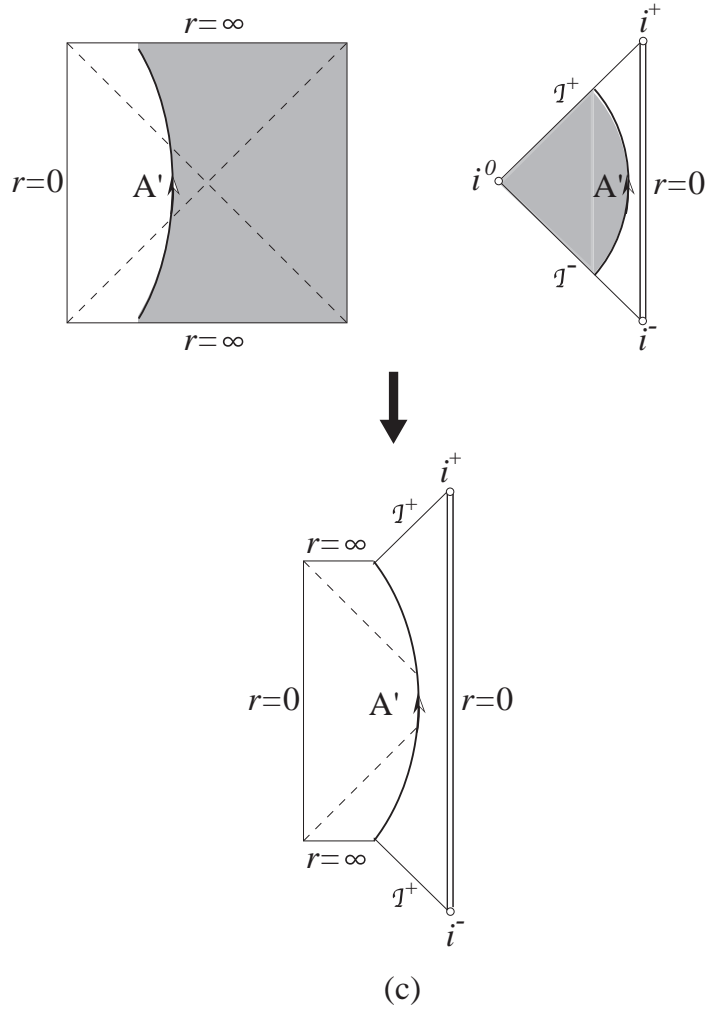
(b)



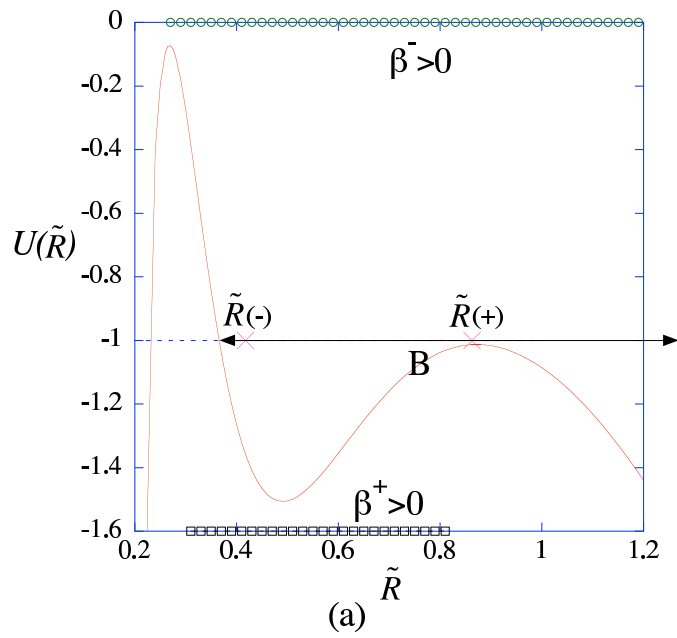
**FIG. 1.** Conformal diagrams for (a) de Sitter spacetime, (b) Reissner-Nordström spacetime with  $m < q$ , and (c) Reissner-Nordström spacetime with  $m > q$ .  $\mathcal{I}^+$  and  $\mathcal{I}^-$  represent future and past null infinity,  $i^+$  and  $i^-$  represent future and past timelike infinity, and  $i^0$  represents spacelike infinity. A double line denotes timelike singularity. In Figs. 2-5 monopole solutions are described by joining a “left” part of de Sitter spacetime to a “right” part of Reissner-Nordström. The “left”  $r = 0$  line in the diagram (a) corresponds to the monopole center. Accordingly, the normal vector  $n^\mu$  of  $\Sigma$  points to the right in all diagrams.  $r_D$  and  $r_{(+)}$  ( $r_{(-)}$ ) denote de Sitter horizons and the black hole outer (inner) horizons, respectively. In the region  $r_- < r_D$  ( $r_+ > r_{(+)}$ ) the sign of  $\beta^- = \partial_n r_-$  ( $\beta^+ = \partial_n r_+$ ) is definite regardless of the shell motion. Brick-pattern domains denote spacetime regions of positive-definite  $\beta$ , while stripe-pattern domains denote those of negative-definite  $\beta$ .

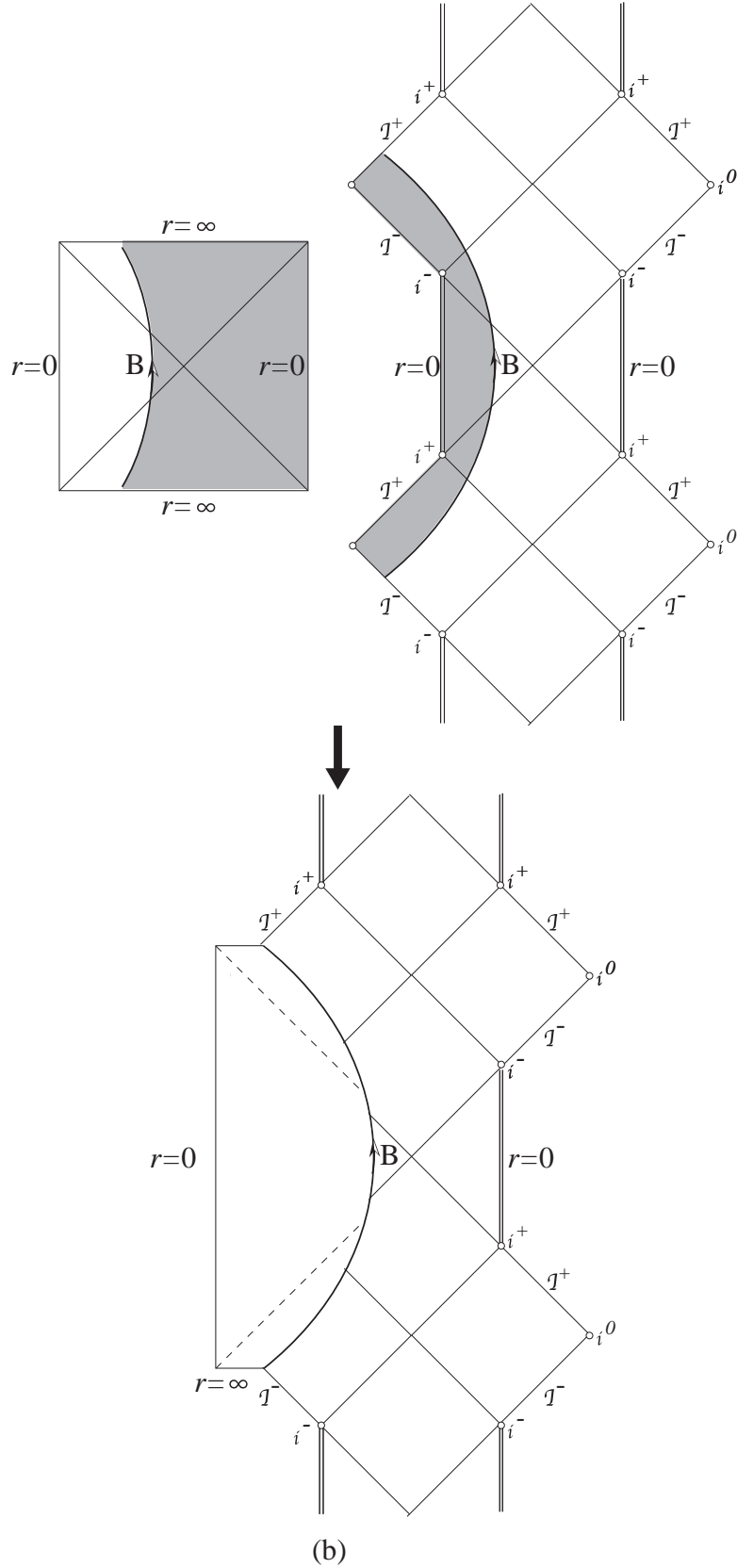




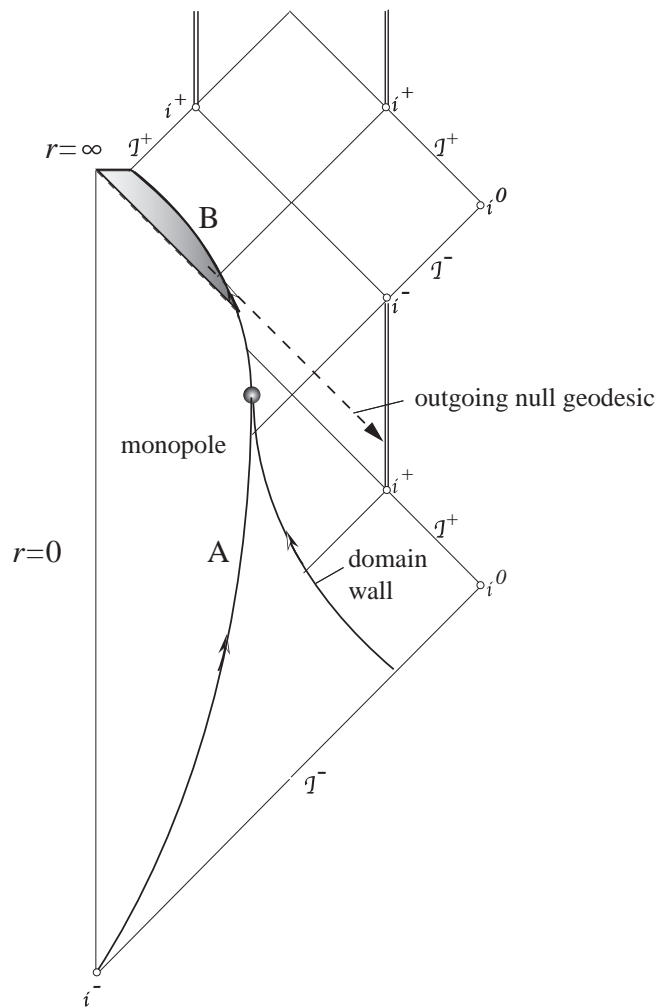
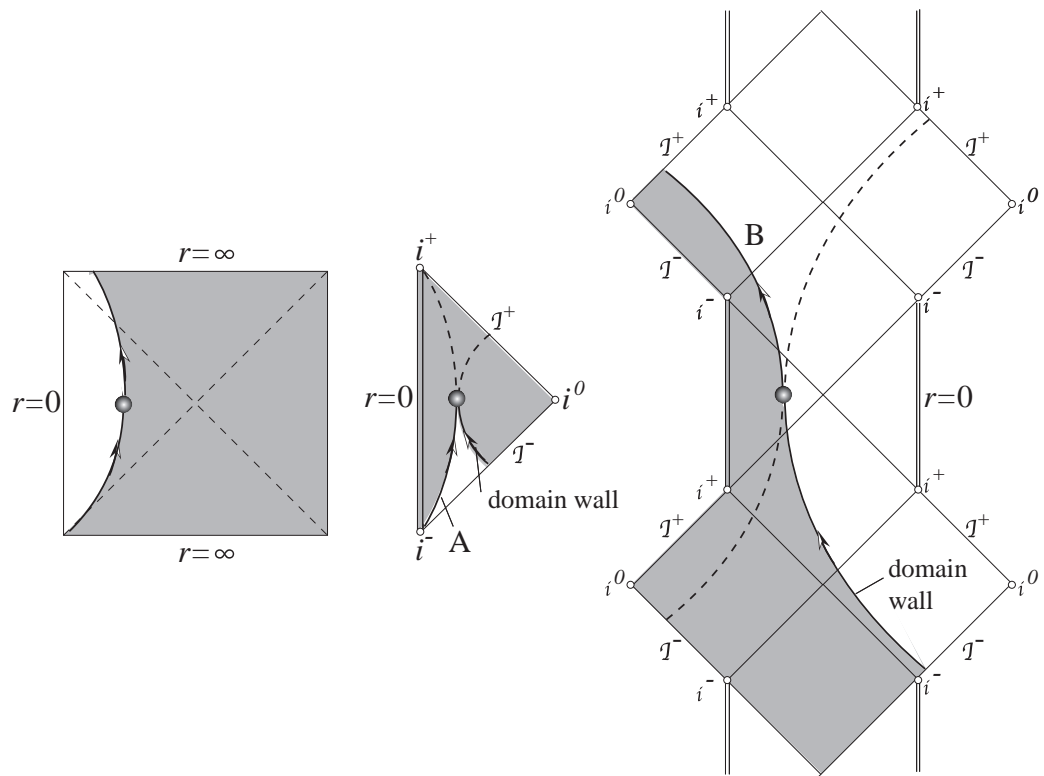


**FIG. 2.** Solutions with  $m = 0.58$ ,  $q = 0.6$ ,  $s_0 = 0.6$  and  $s_1 = 0.1$ . (a) represents the effective potential  $U(\tilde{R})$ . There are a classically stable solution (type A) and a expanding solution (type A'). No black-hole horizon. Circles at the top denote the region of  $\beta^- > 0$ , while squares at the bottom denote the region of  $\beta^+ > 0$ . (b) and (c) show the conformal diagrams of type A and A' solutions, respectively. The upper figures show how the trajectory of the shell is embedded in Reissner-Nordström full spacetime, where gray domains indicate nonexistent regions. The lower figures show complete spacetimes.

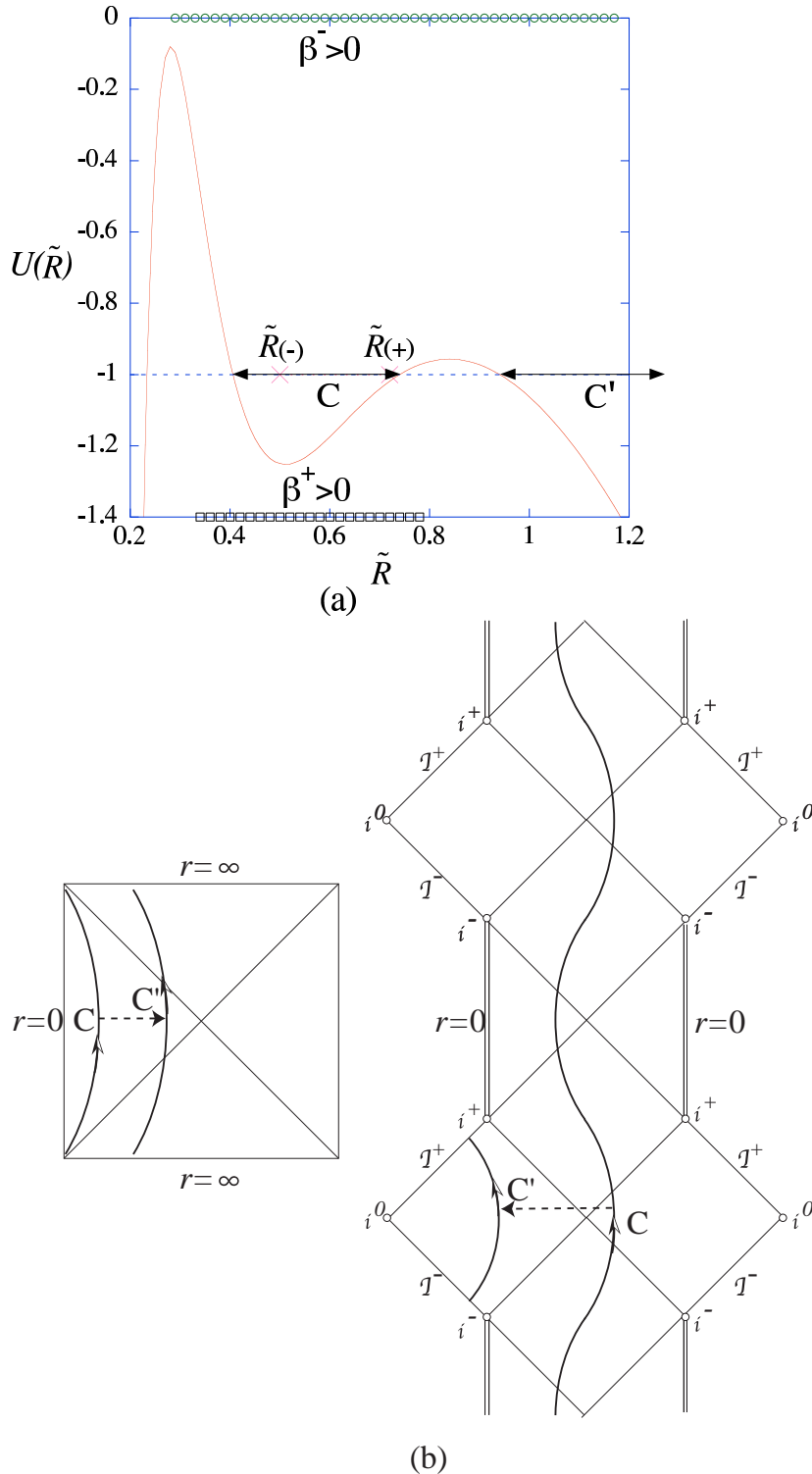




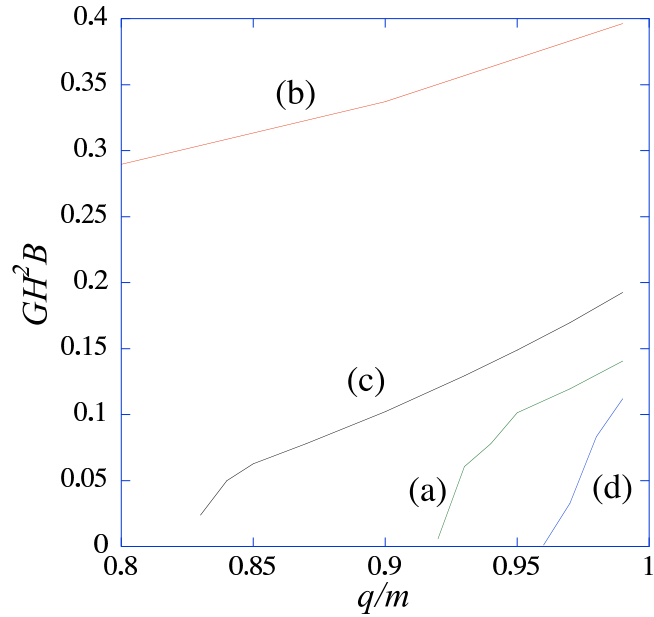
**FIG. 3.** A solution with  $m = 0.64$ ,  $q = 0.6$ ,  $s_0 = 0.6$  and  $s_1 = 0.1$ . (a) represents the effective potential  $U(\tilde{R})$ . There is an inflationary solution (type B). A cross denotes a black-hole horizon. (b) shows the conformal diagram of the solution.



**FIG. 4.** Conformal diagram of the model that a spherical domain wall surrounds the type A monopole and collides with it. A ball denotes an event of the collision. The stable monopole evolves into the type B inflating monopole by the collision. The shaded domain denotes a set of anti-trapped surfaces in the de Sitter side, that is, the created inflationary universe. The past directed outgoing radial null geodesics emanating from this region necessarily hit the timelike singularity.



**FIG. 5.** Solutions with  $m = 0.61$ ,  $q = 0.6$ ,  $s_0 = 0.6$  and  $s_1 = 0.1$ . (a) represents the effective potential  $U(\tilde{R})$ . There are a classically stable solution (type C) and an expanding solution (type C'). (b) shows the conformal diagram of the two solutions. C dashed line denotes a possible tunneling path.



**FIG. 6.** Plot of  $GH^2B$  vs  $q/m$  for several values of  $m$ ,  $s_0$  and  $s_1$ . (a)  $m = 0.6$ ,  $s_0 = 0.6$  and  $s_1 = 0.1$ ; (b)  $m = 0.4$ ,  $s_0 = 0.6$  and  $s_1 = 0.1$ ; (c)  $m = 0.6$ ,  $s_0 = 0.3$  and  $s_1 = 0.1$ ; (d)  $m = 0.6$ ,  $s_0 = 0.6$  and  $s_1 = 0.2$ .